Comparison between Newton-Raphson Method and Fixed-Point Method in Finite Element Analysis with a Vector Hysteresis Model

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The similarities and differences between Newton-Raphson Method (NRM) and Fixed-Point Method (FPM) in finite element computation incorporating a vector Jiles-Atherton hysteresis model is presented in this paper. The importance of the constitutive equation and the differential reluctivity is first introduced. The governing equation is derived based on the constitutive equation, and it is available for both NRM and FPM. The whole expression of Jacobian matrix is derived, which increases the convergence rate significantly when adopting NRM. The system equations when adopting NRM and FPM have the same form if the Jacobian matrix is simplified. The iteration performance of NRM and FPM is tested by computing a current excited magnetic core. The difference on iteration performance is analyzed. The computed results by using NRM and FPM are compared. The detail FEM computation algorithm is not given in the digest due to the page limits. The experimental measured results will be provided and compared with the computed ones in the full paper.

***Index Terms*— Finite Element Method, Fixed-Point Method, Newton-Raphson Method, Vector Hysteresis model.**

I. Introduction

Magnetic hysteresis is one of the natural phenomenon of ferromagnetic materials. Magnetic field computation using numerical techniques, such as, finite element method (FEM) taking account of hysteresis models, can provide more precise approximation to the real situation, such as distorted current waveforms, iron loss, etc. Due to the complexity of material properties, there still exist challenges when embedding the vector hysteresis models directly in the FEM. Several iteration techniques have been investigated and applied to do the hysteretic nonlinear iterations, such as direct iteration method, fixed-point method (FPM), Newton-Raphson method (NRM) [1]-[4]. Whatever iteration method adopted, convergence rate and stability are in the first considerations.

For FPM, the convergence is relative easy to be guaranteed, while the convergence rate is usually slow. For NRM, the Jacobian matrix are calculated from the derivatives of constitutive equation, which increases the risk of divergence. However, the convergence speed of NRM is faster than that of FPM. To speed up the convergence rate of FPM, some techniques are proposed and developed [2]. To guarantee the convergence of NRM, the Jacobian matrix must be calculated in a proper way.

This paper derives the system equations of FEM when using FPM and NRM solving nonlinear magnetic fields incorporating a vector hysteresis model. For both NRM and FPM, the same constitutive equation is adopted. The differential reluctivity is the key issue to ensure the convergence and computation efficiency for both methods. A vector Jiles-Atherton hysteresis model that can describe both isotropic and anisotropic materials is adopted here [5]. It is found that the system equations have the same form for both FPM and NRM when the Jacobian matrix is simplified. The whole expression of the Jacobian matrix is also derived, which is the originality of this paper. By computing the whole Jacobian matrix, the convergence rate will be significantly increased when adopting NRM. Due to the page limits, this part will be given in detail in the full paper.

II. Constitutive Equation

The constitutive law of magnetic materials can be expressed by many kinds of equations. A common one that follows the physical magnetizing principle is as follows,

$$\boldsymbol{B} = \mu_0 \left(\boldsymbol{H} + \boldsymbol{M} \right) \tag{1}$$

where μ_0 is the vacuum permability , $\boldsymbol{B}$ is the magnetic induction vector, $\boldsymbol{H}$ is the magnetic field intensity vector, $\boldsymbol{M}$ is the magnetization of the material. In FEM, however, $\boldsymbol{B}$ is obtained directly from the vector potential during the iteration process, and therefore, a more general expression of the constitutive law is as follow,

$$\boldsymbol{H} = \left[\nu \right] \boldsymbol{B} + \boldsymbol{R}_{\mathrm{M}} \tag{2}$$

where $[\nu]$ is a tensor representing reluctivities which can be constants or variables, and $\boldsymbol{R}_{\mathrm{M}}$ is the remain part that is a function of $\boldsymbol{B}$.

In early publications, $[\nu]$ is usually taken as a constant when adopting FPM [1], mainly considering of the iteration algorithm, and stiffness matrix does not need to be updated during the iterations. Later in some literatures [2][3], $[\nu]$ is treated as variables to speed up the convergence rate of FPM. It can be derived that the convergence rate is most fast when $[\nu]$ equals to the differential reluctivity at the iteration point.

NRM is widely adopted in FEM when dealing with the single valued $\boldsymbol{B}$-$\boldsymbol{H}$ curve. However, when embedding the hysteresis model in FEM, the general equation $\boldsymbol{B}$=$\mu\boldsymbol{H}$ does not work, as $\boldsymbol{B}$ is a multivalued function. It is natural to think of (2) as a alternate expression, and the problem will be how to choose $[\nu]$. In NRM, the derivative of the function is used to find the root, while the whole derivative may not be easily obtainable due to the complexity of governing equations and hysteresis models. Therefore, $[\nu]$ must be set properly to make sure that the Jacobian matrix is close to the derivative of the function. One choice is to used the differential reluctivity, which represents the derivative of the hysteresis loop directly. Furthermore, the differential reluctivity can be obtained from the hysteresis model directly.

III. Field Equations

Substitute the constitutive equation (2) into Maxwell equations, results the following field equations,

$$\nabla \times \left([\nu] \boldsymbol{B} + \boldsymbol{R}_{\mathrm{M}} \right) = \boldsymbol{J}_{\mathrm{s}} \tag{3}$$

where $\boldsymbol{J}_{\mathrm{s}}$ is the current density, $[\nu\ ] = \begin{bmatrix} \nu_{\mathrm{x}} & 0 \\ 0 & \nu_{\mathrm{y}} \end{bmatrix}$ is a tensor, ν_{x}, ν_{y} are made equal to the differential reluctivities of rolling and transverse directions, respectively. The eddy current effect is not included in the formulation. Applying the Galerkin's approximation gives the following equation [6],

$$\int_{\Omega} \left([\nu] \nabla \times \boldsymbol{A} \right) \cdot \left(\nabla \times \boldsymbol{N} \right) d\Omega + \int_{\Omega} \boldsymbol{R}_{\mathrm{M}} \cdot \left(\nabla \times \boldsymbol{N} \right) d\Omega = \int_{\Omega} \boldsymbol{J}_{s} \cdot \boldsymbol{N}\ d\Omega \tag{4}$$

where $\boldsymbol{N}$, and $\boldsymbol{A}$ are the shape function, and magnetic vector potential, respectively. For FPM, with the initial value of $[\nu]$, the magnetic vector potentials are computed, and then $\boldsymbol{R}_{\mathrm{M}}$ on the left of (4) is updated till the computation results satisfy the stop criterion. The new $[\nu]$ will be computed according to the history values of differential reluctivity.

For NRM, the residual $\mathrm{R_i}$ can be directly obtained from (4), and the Jacobian matrix can be derived as follows,

$$\begin{aligned} \mathrm{J_{ij}} = \frac{\partial \mathrm{R_i}}{\partial \mathrm{A_j}} &= \int_{\Omega} [\nu] \left(\nabla \times \boldsymbol{N}_{\mathrm{j}} \right) \cdot \left(\nabla \times \boldsymbol{N}_{\mathrm{i}} \right) d\Omega \\ &+ \int_{\Omega} \frac{\partial [\nu]}{\partial \mathrm{A_j}} \left(\nabla \times \boldsymbol{A} \right) \cdot \left(\nabla \times \boldsymbol{N}_{\mathrm{i}} \right) d\Omega + \int_{\Omega} \frac{\partial \boldsymbol{R}_{\mathrm{M}}}{\partial \mathrm{A_j}} \cdot \left(\nabla \times \boldsymbol{N}_{\mathrm{i}} \right) d\Omega. \end{aligned} \tag{5}$$

If only the first item in (5) is taken into account, the system equations using NRM are same with the ones when adopting FPM. The only difference will be how $[\nu]$ is computed. For FPM, $[\nu]$ is computed from the history values of differential reluctivity, while for NRM, $[\nu]$ is computed from the current time step. The differential reluctivity is computed from the field values of adjacent time steps. The Jacobian matrix and the residual are updated for each iteration. The computation will move to next time step till the residual error is smaller than the criterion value.

IV. Comparison of Computed Results

The FEM program is developed based on the above governing equations and the vector hysteresis model. Both NRM and FPM are implemented. The detail computation algorithm will be introduced in the full paper. The proposed algorithm is implemented to calculate the magnetic field of a rectangular magnetic core with a sinusoidal current excitation. Fig. 1 shows the comparison of iterations in one period between FPM and NRM. Generally speaking, the convergence rate of NRM is faster than FPM. The computed B locus by both NRM and FPM in one element is shown in Fig. 2. The detail analysis and experiment results will be given in the fullpaper.

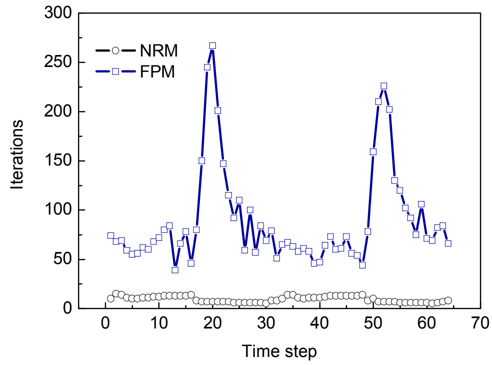

Fig. 1. The statistics of iterations of NRM and FPM.

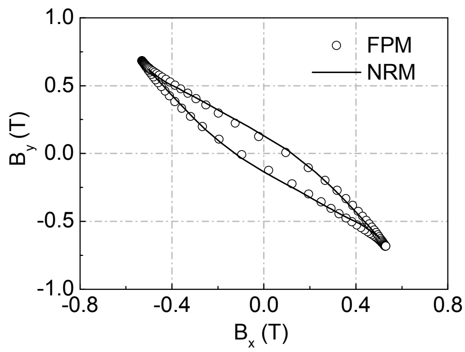

Fig. 2. The computed $\boldsymbol{B}$ loci by NRM and FPM respectively.

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